Cosmological Constraints on an Invisibly Decaying Higgs

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Working in the context of a proposal for collisional dark matter, we derive bounds on the Higgs boson coupling g' to a stable light scalar particle, which we refer to as phion (ϕ), required to solve problems with small scale structure formation which arise in collisionless dark matter models. We discuss the behaviour of the phion in the early universe for different ranges of its mass. We find that a phion in the mass range of 100 MeV is excluded and that a phion in the mass range of 1 GeV requires a large coupling constant, $g' \gtrsim 2$, and $m_h \lesssim 130$ GeV in order to avoid overabundance, in which case the invisible decay mode of the Higgs boson would be dominant.

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I. INTRODUCTION

We have recently discussed the role of a light, stable, strongly self-coupled scalar field in solving the problems of the Cold Dark Matter (CDM) model for structure formation in the Universe, concerning galactic scales [1]. Our proposal involves a particle physics-motivated model, where the DM particles are allowed to self-interact so as to have a large scattering cross section and negligible annihilation or dissipation. The self-interaction results in a characteristic length scale given by the mean free path of the particle in the halo. This idea was originally proposed to suppress small scale power in the standard CDM model [2,3] and has been recently revived, in a general context, in order to address the issues discussed above [4]. Our model [1] is a concrete realization of this idea, which involves an extra gauge singlet as the self-interacting, non-dissipative cold dark matter particle. Following Ref. [5], we call this scalar particle phion, ϕ, and assume that it couples to the Standard Model (SM) Higgs boson, h, with a Lagrangian density given by:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_\phi^2\phi^2 - \frac{g}{4!}\phi^4 + g' v \phi^2 h \quad , \tag{1}$$

where g is the phion self-coupling constant, m_ϕ its mass, $v = 246$ GeV is the Higgs vacuum expectation value and g' is the coupling between ϕ and h. A model along these lines have been previously discussed [6]. Clearly the interaction term between the phion and the Higgs boson arises from a quartic interaction $\frac{g'}{2}\phi^2 H^2$, where H is the electroweak Higgs doublet. As shown in [1], the ϕ mass does not arise from spontaneous symmetry breaking since this would yield a tiny scalar self-coupling constant. The phion mass in (1) should be regarded as as a phenomenological parameter arising from a more encompassing theory.

As is well known, scalar particles have been repeatedly invoked as DM candidates [7–12]; however, our proposal has the salient feature that it brings about a connection with the SM Higgs boson which could arise in extensions of the SM. For instance, the hidden sector of heterotic string theories does give rise to astrophysically interesting self-interacting scalars [13]. For reasonable values of g', the new scalar would introduce a novel invisible decay mode for the Higgs boson. This could, in principle, provide an explanation for the failure in finding the Higgs boson at accelerators sofar [14], which will be tested at future colliders.

On the astrophysical front, recent observational data on large scale structure, cosmic microwave background anisotropies and type Ia supernovae suggest that $\Omega_{tot} \approx 1$, of which $\Omega_{baryons} \approx 0.05$ and $\Omega_\Lambda \approx 0.65$ [15]; the remaining contribution, $\Omega_{DM} \approx 0.3$ (apart from neutrinos that may contribute a small fraction), comes from dark matter (DM), which determines the hierarchy of the structure formation in the Universe. The most prominent theories of structure formation are now ΛCDM and QCDM, which consist, respectively, of the standard Cold Dark Matter (CDM) model supplemented by a cosmological constant or a dark energy, i.e. a negative pressure component.

In the CDM model, initial Gaussian density fluctuations, mostly in non-relativistic collisionless particles, the so-called cold dark matter, grow during the inflationary period of the Universe and evolve, via gravitational

instability, into the structures one observes at present. However, it has been found that the CDM model cannot sucessfully accomodate the data observed on all scales. For instance, N-body simulations predict a number of halos which is a factor ~ 10 larger than the observed number at the level of Local Group [16,17]. Furthermore, astrophysical systems that are DM dominated, e.g. dwarf galaxies [18–20] and low surface brightness galaxies [21] show shallow matter–density profiles with finite central densities. This is in contradiction with galactic and galaxy cluster halos in high resolution N-body simulations [22–25], which have singular cores, with $\rho \sim r^{-\gamma}$ and γ in the range between 1 and 2. This can be understood as cold collisionless DM particles do not have any characteristic length scale leading, due to hierarchical gravitational collapse, to dense dark matter halos with negligible core radius.

It is relevant to stress that recent numerical simulations [26–29] indicate that self-interaction of DM particles does bring noticeable improvements on properties of the CDM model on small scales. We point out, however, that a numerical simulation that takes into account the salient features of our proposal is still missing.

At present, ϕ particles are non-relativistic, with typical velocities $v \simeq 200$ km s^{-1}, and, therefore, it is impossible to dissipate energy creating more particles in reactions like $\phi\phi \to \phi\phi\phi\phi$. Thus, as only the elastic channel is kinematically accessible, near threshold, the cross section is given by:

$$\sigma(\phi\phi \to \phi\phi) \equiv \sigma_{\phi\phi} = \frac{g^2}{16\pi s} \simeq \frac{g^2}{64\pi m_\phi^2} \quad . \tag{2}$$

A limit on m_ϕ and g can be obtained by demanding that the mean free path of the particle ϕ, λ_ϕ, is in the interval 1 kpc $< \lambda_\phi <$ 1 Mpc [4]. Hence, we have:

$$\lambda_\phi = \frac{1}{\sigma_{\phi\phi} n_\phi} = \frac{m_\phi}{\sigma_{\phi\phi}\rho_\phi^h} \quad , \tag{3}$$

where n_ϕ and ρ_ϕ^h are, respectively, the number and mass density of ϕ particles in the halo. Eqs. (2) and (3) imply:

$$\sigma_{\phi\phi} = 2.1 \times 10^3 \left(\frac{m_\phi}{\text{GeV}}\right)\left(\frac{\lambda_\phi}{\text{Mpc}}\right)^{-1} \times \left(\frac{\rho_\phi^h}{0.4\ \text{GeVcm}^{-3}}\right)^{-1} \ \text{GeV}^{-2} \quad , \tag{4}$$

which, in turn, leads to:

$$m_\phi = 13\ g^{2/3}\left(\frac{\lambda_\phi}{\text{Mpc}}\right)^{1/3}\left(\frac{\rho_\phi^h}{0.4\ \text{GeVcm}^{-3}}\right)^{1/3} \text{MeV} \quad . \tag{5}$$

In what follows, we shall see how the requirement that $\Omega_\phi h^2 \simeq 0.3$, i.e. that the phion is a suitable DM candidate, and that the phion is able to explain small scale structure, leads to bounds on the couplings g and g'.

II. PHION DENSITY ESTIMATE

In the ensuing discussion, we shall bear in mind the most recent lower bound on the mass of the SM Higgs boson, as it emerges from the combined LEP data gathered at energies up to 205.9 GeV, $m_h > 112.3$ GeV at 95% confidence level [30]. On the other hand, SM precision data indicates that $m_h < 188$ GeV at 95% confidence level [30] and $m_h \leq 306$ GeV at 99% confidence level [31].

If g' is sufficiently small, phions decouple early in the thermal history of the Universe and are diluted by subsequent entropy production. In Ref. [1], we have considered out-of-equilibrium phion production via inflaton decay in the context of $N = 1$ Supergravity inflationary models (see. e.g. [32] and references therein).

On the other hand, for certain values of the coupling g', it is possible that ϕ particles are in thermal equilibrium with ordinary matter. In order to determine whether this is the case, we will make the usual comparison between the thermalization rate Γ_{th} and the expansion rate of the Universe H.

The thermalization rate is given by

$$\Gamma_{th} = n < \sigma_{ann} v_{rel} > \quad , \tag{6}$$

where $n = 1.2 \times T^3/\pi^2$ is the density of relativistic phions and $< \sigma_{ann} v_{rel} >$ is the annihilation cross section averaged over relative velocities. On the other hand, the expansion rate is given by:

$$H = \left(\frac{4\pi^3 g_*}{45}\right)^{1/2} \frac{T^2}{M_P} = 1.66 \times g_*^{1/2} \frac{T^2}{M_P} \quad . \tag{7}$$

At temperatures above the electroweak phase transition, $T_{EW} \simeq 300$ GeV, a typical value in many extensions of the SM where one hopes to find the required features to achieve successful baryogenesis [33,34], the order parameter (the vacuum expectation value of the Higgs field) vanishes, $v(T) = 0$, and therefore the $\phi\phi h$ coupling is non-operative. However, as mentioned above, this interaction term has its origin in the 4-point coupling, $\phi\phi hh$, which can bring, at high temperatures, the phion-Higgs system into thermal equilibrium. Using the temperature as the center-of-mass energy, the cross section is given by:

$$\sigma_{ann} v_{rel} \simeq \frac{g'^2}{32\pi T^2} \quad , \tag{8}$$

which implies that phions are in thermal equilibrium for temperatures smaller than

$$T_{eq} \simeq \frac{g'^2 M_P}{32\pi^3 g_*^{1/2}} \quad . \tag{9}$$

Therefore, phions will never be in thermal equilibrium before the electroweak phase transition if $g' \lesssim 10^{-7}$.

Thermal equilibrium can be achieved just below T_{EW}, when the trilinear coupling is operative, if g' is such that

the thermalization rate, Γ_{th}, is larger than the Hubble expansion rate. We now compute the required bounds on g'.

The phion annihilation cross section at high energies ($T \gtrsim m_h$) is given by a relativistic Breit-Wigner resonance formula:

$$\sigma_{ann} v_{rel} = \frac{4\pi(s/m_h^2)\Gamma(h \to \phi\phi)\Gamma_h}{(s-m_h^2)^2 + m_h^2\Gamma_h^2} \quad , \tag{10}$$

where Γ_h is the total Higgs decay rate. At the resonance peak ($s = m_h^2$) it becomes

$$\sigma_{ann} v_{rel} = \frac{4\pi}{m_h^2} BR(h \to \phi\phi) \quad . \tag{11}$$

Using

$$\Gamma(h \to \phi\phi) = \frac{g'^2 v^2 (m_h^2 - 4m_\phi^2)^{1/2}}{32\pi m_h^2} \quad , \tag{12}$$

we obtain the decoupling temperature in the limit $m_h \gg m_\phi$

$$T_D \simeq 150 \frac{\Gamma_h m_h^3}{g'^2 M_P v^2} \quad . \tag{13}$$

This implies that, in order to have a decoupling temperature of the order of the Higgs mass, the coupling constant should be fairly small:

$$g' \simeq 10^{-10} \quad , \tag{14}$$

where we have introduced the SM value of $\Gamma_h = 3.2$ MeV, obtained from the code HDECAY [35], used to compute the width of a 115 GeV Higgs boson.

Therefore, if $g' \geq 10^{-10}$, the phions will be brought into thermal equilibrium right after the electroweak phase transition. If this is the case, there are two possible scenarios depending whether they decouple while relativistic or otherwise.

In order to study these scenarios, we need to determine the decoupling temperature at $T \simeq m_\phi \ll T_{EW}$, in which case the phion annihilation cross section involves virtual Higgs exchange (h^*), as in Figure 1, and is given by [36]:

$$\sigma_{ann} v_{rel} = \frac{8 g'^2 v^2}{(4m_\phi^2 - m_h^2)^2 + m_h^2 \Gamma_h^2} F_X \quad , \tag{15}$$

where

$$F_X = \lim_{m_{h^*} \to 2m_\phi} \left(\frac{\Gamma_{h^* X}}{m_{h^*}} \right) \quad , \tag{16}$$

and $\Gamma_{h^* X}$ denotes the width for the decay $h^* \to X$ ($X \neq \phi\phi$, since we are dealing with inelastic scattering only), for $m_{h^*} = 2m_\phi$. For the mass range of interest to us, $m_\phi \sim 10 - 100$ MeV, one has $F_X \sim 10^{-13}$ [37].

In this case, the relationship between the coupling constant g' and the decoupling temperature T_D is given by

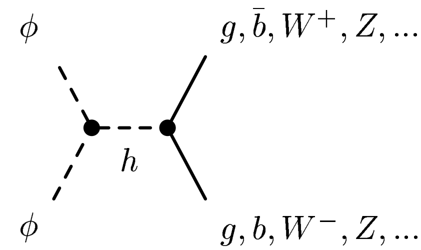

FIG. 1. Feynman diagramm for phion annihilation via Higgs exchange.

$$g'^2 = 5.5 \frac{(m_h/100\text{GeV})^4}{(T_D/\text{MeV})} \quad . \tag{17}$$

If $g' \leq 0.1$, the phions decouple while relativistic and are as abundant as photons. Since we are interested in stable light phions, it is a major concern avoiding phion overproduction if it decouples while relativistic. In fact, in this case, there is an analogue of Lee-Weinberg limit for neutrinos (see e.g. [38]):

$$\Omega_\phi h^2 \simeq 0.08 \frac{m_\phi}{1 \text{ eV}} \quad , \tag{18}$$

which results in a very stringent bound, $m_\phi \lesssim 4$ eV, and implies that the phion self-coupling constant should satisfy $g \lesssim 2.5 \times 10^{-10}$, in order to solve the small scale structure problem that exists with conventional collisionless cold dark matter.

In order that the phions decouple non-relativistically and their abundance reduces to acceptable levels without fine-tuning the self-coupling constant, $g' > 0.1$ is required. In this case, using standard methods to compute the phion relic abundance [36,38], one obtains:

$$\Omega_\phi h^2 = \frac{1.07 \times 10^9 x_F}{g_*^{1/2} M_P \langle \sigma_{ann} v_{rel} \rangle} \quad , \tag{19}$$

where g_* denotes the number of degrees of freedom in equilibrium at annihilation and $x_F \equiv m_\phi/T_F$ is the inverse of the freeze-out temperature in units of the phion mass. The relevant crosss section is the phion annihilation cross section involving virtual Higgs exchange, Eq. (15), with $F_X \sim 10^{-13}$ [37]. The freeze-out temperature is determined by the solution of the equation

$$x_F \simeq \ln[0.038 (g_* x_F)^{-1/2} M_P m_\phi \langle \sigma_{ann} v_{rel} \rangle] \quad . \tag{20}$$

In order to get $x_F \geq 1$ and apply Eq. (19), one must have $g' \gtrsim 1.2$, for $m_\phi = 50$ MeV and $m_h = 115$ GeV. However, due to the smallness of the cross section, the relic abundance of the phion is several orders of magnitude larger than the observed value. Hence, one concludes that a phion in this mass range is excluded.

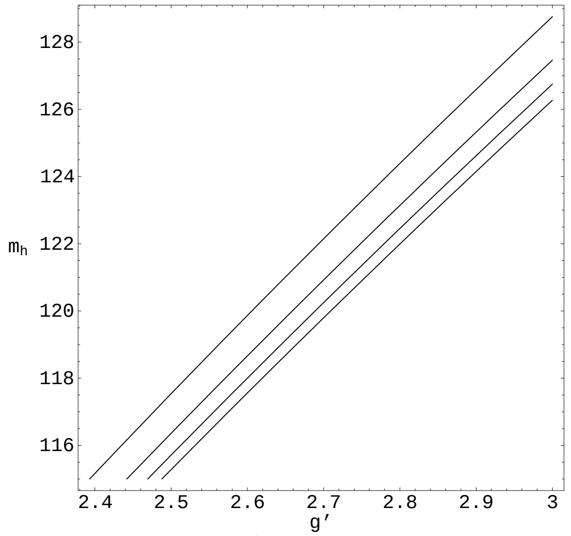

FIG. 2. Contour of $\Omega_\phi h^2 = 0.3$ as a function of m_h (in GeV) and g', for $m_\phi = 0.5$ GeV (top), 1.0, 1.5 and 2 GeV (bottom).

The smallness of the phion annihilation cross section for $m_\phi \simeq 50$ MeV is due to the small factor $F_X \simeq 10^{-13}$. However, this factor increases significantly with larger phion masses. Considering $m_\phi \simeq 1$ GeV, then it follows that $F_X \simeq 10^{-7}$. We find that the requirement $\Omega_\phi h^2 \simeq 0.3$ implies that $m_\phi \gtrsim 500$ MeV and $g' \gtrsim 2$, a solution which holds only for $m_h \lesssim 130$ GeV. Heavier phion and Higgs particles tend to make $\Omega_\phi h^2 > 0.3$. We have verified that our solutions respect the condition $x_F > 3$, meaning that the phion behaves as a cold dark matter candidate. Of course, our solutions are at the edge of validity of perturbation theory. Our results are depicted in Figure 2.

For these large values of the coupling constant, the decay width of the Higgs into phions is given by:

$$\Gamma(h \to \phi\phi) = 5.23 \left(\frac{m_h}{115 \text{ GeV}}\right)^{-1} g'^2 \text{ GeV}. \qquad (21)$$

Hence, in this case the Higgs width is totally dominated by the invisible decay mode and this model can be easily tested at future colliders (see e.g. [39]).

III. DISCUSSION AND CONCLUSIONS

In this work, we have derived, from fairly general cosmological arguments, bounds on g', the coupling constant of the Higgs boson to stable scalar particles, which contribute to Higgs decay via invisible channels. These particles, the phions, have all the required features to be regarded a successful self-interacting dark matter candidate and solve the difficulties of the CDM model on small scales.

We have found that, for $g' \lesssim 10^{-10}$, the phions never reach thermal equilibrium and hence can only be produced by out-of-equilibrium decay of the inflaton field, as discussed in [1]. In this case, the phion does not contribute to the invisible Higgs boson decay channel. For $g' \gtrsim 10^{-10}$, we have found that, if $g' \lesssim 0.1$, the phion decouples while still relativistic and a limit for its mass, $m_\phi \lesssim 4$ eV can be derived, which, in turn, implies in a strong bound on the phion self-coupling constant, $g \lesssim 10^{-9}$, if the phion is required to solve the CDM model problems on small scales. On the other hand, if $g' \gtrsim 1$, the phion decouples while non-relativistic; however, its abundance is not cosmologically acceptable for phion masses in the range of $50 - 100$ MeV due to the small annihilation cross section. For masses in the range of $0.5 - 2$ GeV, we have found that abundances of $\Omega_\phi h^2 \simeq 0.3$ require large values of the coupling $g' \simeq 2.5$ and $m_h \lesssim 130$ GeV. In this scenario, the Higgs width is dominated by the invisible $h \to \phi\phi$ mode and can be tested at future colliders.

Regarding the origin of the phion mass, it is possible that it arises from the cancellation between a tachyonic mass, due to the dynamics of fields in a more encompassing theory together with the contribution from the quartic interaction, $g'^2 v^2$. In fact, in case the only relevant fields are the phion and the Higgs boson, the phion mass is dominated by a term which arises from electroweak symmetry breaking, namely, $m_\phi \simeq g' v$, and if the phion is to solve the small structure problem, this would imply $g' \simeq 5 \times 10^{-5}$ which, in turn, brings us to the overproduction problem.

Finally, we comment on the recent observation [40] that CDM problems at small scales can be fixed only through s-channel annihilation of DM particles, that is $\sigma_{ann} v_{rel} \propto const$ so that $\sigma_{ann} v_{rel} = 2.5 \times 10^{-1} (m/\text{GeV})\ \text{GeV}^{-2}$. We have verified that these conditions cannot be met in our model. On more general grounds, it is difficult to see how substantial amounts of annihilating DM could be obtained from any processes in the early Universe, given that the required cross section is so large, although it is clearly a quite interesting challenge to build concrete particle physics models that exhibit such features.

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